Uncertainty and Learning in Pharmaceutical Demand

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Motivation

- How doctors prescribe a sequence of drugs under uncertainty?
- Uncertainty: heterogeneity in patients' illnesses and drugs' effects.
- Bayesian agents (patients and doctors) learn from prescription experience.

Research objective

- Measuring the effects of uncertainty and learning on prescription choices and treatment outcomes.

Preview of the Main Results

- Substantial heterogeneity in drugs' effects across patients.
- Strong evidence of learning: there are reductions in uncertainty after even a single prescription.
- This reduction leads to persistence in drug choices.
- Therefore, learning enables agents to reduce the costs of uncertainty.

Italian Anti-ulcer Market

- Patients' medical costs are covered by a National Health System
  → No variation in insurance status across patients
  → Doctros face a uniform incentive scheme.
- Anti-ulcer market is almost entirely drug-based
  → No concern about choosing expensive treatment procedures.
- Drug prices are set by the regulatory Drug Commission
  → Drugs of therapeutic equivalence are assigned the same price

Italian Anti-ulcer Market

TABLE I

THE ITALIAN ANTI-ULCER MARKET

Summary Statistics from the Data

#	Molecule	Patent-Holder	In-Sample Mkt. Share[c]	Major Brands[a] in Italian Mkt.	Date of Entry	Avg.[b] Price
1	Ranitidine	Glaxo	64.4	Zantac*, Ranidil	1981	$2.90
2	Omeprazole	Astra	11.0	Losec*, Omeprazen	1990	$3.14
3	Famotidine	Merck	6.8	Pepcid*, Famodil	1986	$2.59
4	Nizatidine	Lilly	3.2	Axid*, Zanizal	1988	$2.74
5	19 others	—	14.6	Various	<1981	$1.42[d]

Data

- Patient-level monthly anti-ulcer prescriptions data between January 1990 and December 1992.
- Include only patients who are first observed after June 1990 to avoid left-censoring.
- Patients receive, on average, 2.8 prescriptions for 1.2 drugs over a period of under 6 months.

Overview of Model

- Doctor performs an initial diagnostics and selects an initial drug treatment.
- Conditional on the initial diagnosis, each doctor has uncertainty about the effectiveness of various drug alternatives.
- This uncertainty is modeled by patient-specific "match values" associated with each drug.
- "Match values" have two dimension: *symptomatic* effect and *curative* effect

  Discussion: *Curative* effect?

Model

Each doctor is forward-looking and selects the sequence of drugs that maximizes her patient's expected utility:

$$\max_{D=\{\{d_{jnt}\}_{n=1}^{N}\}_{t=1}^{\infty}} E_D \sum_{t=1}^{\infty} \beta^t d_{int} u_{jnt} (1 - \omega_{j,t-1})$$

d_{jnt} : 1 if patient j takes drug n in period t

u_{jnt} : Single-period utility flow

w_{jt} : 1 if patent j recovers after period t

β : Discount factor $\in (0, 1)$

The doctor chooses the sequence D to maximizes the expected utility.

Model

The specification for patient j's single-period utility is Constant Absolute Risk Aversion function:

$$u(x_{jnt}, p_n, \epsilon_{jnt}) = -exp(-r * x_{jnt}) - \alpha * p_n + \epsilon_{jnt}$$

x_{jnt} : Patient j's symptomatic signal from taking drug n in period t

p_n : Per-prescription price of drug n

ϵ_{jnt} : Idiosyncratic error

r : Degree of risk aversion(> 0)

The doctor chooses the sequence D to maximizes the expected utility.

Model

Let h_{jt} denote the probability that patent j recovers by the end of period t:

$$h_{jt}\left(h_{jt-1}, y_{jnt}\right) = \frac{\left(\frac{h_{jt-1}}{1-h_{jt-1}}\right) + d_{jnt}y_{jnt}}{1 + \left(\left(\frac{h_{jt-1}}{1-h_{jt-1}}\right) + d_{jnt}y_{jnt}\right)}$$

where y_{jnt} is the curative signal and the initial condition h_{0j} is defined

$$h_{0j} = \theta_k \quad \text{w.p. } p_k, \quad k = 1, \ldots, K$$

where $0 < p_i < 1, \sum p_i = 1$, and $0 \leqslant \theta_1, \ldots, \theta_K \leqslant 1$.
h_{0j} is refereed as patient j's "initial illness severity".

Learning Process

Doctors begin treatment with prior information of two match values, μ_{jn} and ν_{jn}, the symptomatic and curative effects of the drug.

$$\begin{pmatrix} \mu_{jn} \\ \nu_{jn} \end{pmatrix} \sim N\left(\begin{bmatrix} \underline{\mu}_{nk} \\ \underline{\nu}_{nk} \end{bmatrix}, \begin{bmatrix} \underline{\sigma}_n^2 & 0 \\ 0 & \underline{\tau}_n^2 \end{bmatrix}\right)$$

Patient j does not know μ_{jn} and ν_{jn}, but she receives signals which allow her to update.

$$\begin{pmatrix} x_{jnt} \\ y_{jnt} \end{pmatrix} \sim N\left(\begin{bmatrix} \mu_{jn} \\ \nu_{jn} \end{bmatrix}, \begin{bmatrix} \sigma_n^2 & 0 \\ 0 & \tau_n^2 \end{bmatrix}\right)$$

Discussion: Zero correlation?

Learning Process

Let I_{jn}^t denote the number of times that patient j has taken drug n up to period t. Patient j's posterior beliefs regarding μ_{jn} are

$$\mu_{jn}^{t+1} = \begin{cases} \dfrac{\frac{\mu_{jn}^t}{V_{jn}^t} + \frac{x_{jnt+1}}{\sigma_n^2}}{\frac{1}{V_{jn}^t} + \frac{1}{\sigma_n^2}} = \dfrac{\sigma_n^2 \mu_{jn}^t + V_{jn}^t x_{jnt+1}}{\sigma_n^2 + V_{jn}^t} & \text{if drug } n \text{ taken in period } t+1, \\ \mu_{jn}^t, \quad \text{otherwise,} \end{cases}$$

$$V_{jn}^{t+1} = \begin{cases} \dfrac{1}{\frac{1}{\sigma_n^2} + \frac{I_{in}^{t+1}}{\sigma_n^2}} = \dfrac{\sigma_n^2 \sigma_n^2}{\sigma_n^2 + I_{jn}^{t+1} \sigma_n^2}, & \text{if drug } n \text{ taken in period } t+1, \\ V_{jn}^t, & \text{otherwise.} \end{cases}$$

Dynamic Drug Choice

State Variables :

(1) Patient j's posterior mean match values μ_{jn}^t and ν_{jn}^t.
(2) Counts of the number of times that patient j has tried each drug, I_{jn}^t
(3) Recovery probability h_{jt}
(4) Idiosyncratic errors ϵ_{jnt}

The transition rules are given in the previous slides.

Dynamic Drug Choice

Recall the maximization problem:

$$\max_{D=\{\{d_{jnt}\}_{n=1}^{N}\}_{t=1}^{\infty}} E_D \sum_{t=1}^{\infty} \beta^t d_{int} u_{jnt} (1 - \omega_{j,t-1})$$

The value function $W(S_t)$ in the infinite-horizon problem is defined via Bellman equation,

$$\begin{aligned}
W(S_t) &= \max_n E[u(x_{jnt}, p_n, \epsilon_{jnt}) + \beta(1 - \omega_{jt}) E[W(S_{t+1}|x_{jnt}, y_{jnt}, n]|S_t] \\
&= \max_n E[u(x_{jnt}, p_n, \epsilon_{jnt}) + \beta(1 - E[\omega_{jt}|y_{jnt}]) E[W(S_{t+1}|x_{jnt}, y_{jnt}, n]|S_t] \\
&= \max_n E[u(x_{jnt}, p_n, \epsilon_{jnt}) + \\
&\qquad \beta(1 - h_{jt}(h_{jt-1}, y_{jnt})) E[W(S_{t+1}|x_{jnt}, y_{jnt}, n]|S_t] \\
&= \max_n E[-\exp(-r\mu_{jn}^t + \frac{1}{2} r^2 (\sigma_n^2 + V_{jn}^t)) - \alpha p_n + \epsilon_{jnt} + \\
&\qquad \beta(1 - h_{jt}(h_{jt-1}, y_{jnt})) E[W(S_{t+1}|x_{jnt}, y_{jnt}, n]|S_t]
\end{aligned}$$

Estimation Results

TABLE III

DYNAMIC MODEL: PARAMETER ESTIMATES

Parameter	Est.	Std. Err.	Est.	Std. Err.
Illness heterogeneity distribution	Recovery Probability		Type Probability	
θ_1 (Type 1)	0.433	0.003	0.593	0.006
θ_2 (Type 2)	0.127	0.003	0.335	0.006
θ_3 (Type 3)	0.199	0.007	0.043	0.001
θ_4 (Type 4)	0.432	0.011	0.029	0.002
Means, symptom match values[b]	Type 1		Type 2	
$\underline{\mu}_1$	0.927	0.282	1.195	0.369
$\underline{\mu}_2$[c]	0.928	0.287	0.428	0.166
$\underline{\mu}_3$	0.481	0.197	−0.028	0.178
$\underline{\mu}_4$	0.335	0.161	−0.145	0.079
$\underline{\mu}_5$	0.451	0.174	−0.483	0.137
Means, curative match values[b]	Type 1		Type 2	
$\underline{\nu}_1$	0.014	0.003	0.006	0.000
$\underline{\nu}_2$[c]	0.015	0.005	0.006	0.001
$\underline{\nu}_3$	0.013	0.030	0.006	0.095
$\underline{\nu}_4$	0.013	0.084	0.014	0.009
$\underline{\nu}_5$	−0.034	0.000	−0.038	0.000
Std. dev., symptom match values				
$\underline{\sigma}$	1.574	0.448		
Std. devs., symptom signals				
σ_1	0.998	0.287		
σ_2	1.134	0.326		
σ_3	1.375	0.395		
σ_4	1.159	0.333		
σ_5	0.931	0.268		
Std. dev., curative match values				
$\underline{\tau}$	0.007	0.000		
Std. dev., curative signals				
τ	0.007	0.001		
Price coefficient, α[a]	1.080	0.091		
Risk-aversion parameter, r	0.990	0.274		
Discount rate, β	0.950	Fixed		
Number of observations	34,972			
Number of similar draws	30			
Log likelihood function	−124,484.34			

Importance of Uncertainty and Learning

TABLE VI

RESULTS FROM COUNTERFACTUAL SIMULATIONS

Baseline Specification[a]	
Avg. discounted utility	−28.7
Avg. treatment length	4.8
Avg. treatment cost	245
Avg. number of different drugs	1.4
Market shares	
Drug 1	60.4
Drug 2	14.1
Drug 3	3.7
Drug 4	2.5
Drug 5	19.3
Herfindahl index	4,242
Counterfactual I: Complete Information[b]	
Avg. discounted utility	−26.4
Avg. treatment length	8.8
Avg. treatment cost	385
Avg. number of different drugs	1.9
Market shares	
Drug 1	22.4
Drug 2	12.9
Drug 3	12.0
Drug 4	10.9
Drug 5	41.8
Herfindahl index	2,676
Counterfactual II: No Experimentation[c]	
Avg. discounted utility	−30.6
Avg. treatment length	4.8
Avg. treatment cost	248

Discussion

- Policy Implication
- Generalization